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TWO COUNTRIES: INTERNATIONAL UNDEREMPLOYMENT
INFLATIONARY STEADY-STATE UNBALANCE GROWTH

Hans Brems, Professor, Department of Economics

#523

College of Commerce and Business Administration
University of Illinois at Urbana-Champaign

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Summary:

In a two-country neoclassical growth model with a flexible exchange rate simple solutions are found for underemployment inflationary steady-state equilibrium growth. Each rate of growth of the four real wage rates will be the same for any value of the employment fraction and for any value of the inflationary potential. As a result, Friedman's "natural" rate of unemployment is not unique. Employment and inflation cannot both be controlled by monetary policy. International growth will be balanced only as an odd piece of luck. The purchasing power parity theory of the exchange rate is not the whole story: The rate of growth of the exchange rate depends upon the rates of growth of physical outputs no less than upon the rates of growth of prices. Ultimately it is found to depend upon the employment fractions and the inflationary potentials of both countries.

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TWO COUNTRIES: INTERNATIONAL UNDEREMPLOYMENT INFLATIONARY STEADY-STATE UNBALANCED GROWTH

By HANS BREMS

The purpose of the present chapter is to open our closed economy and study international growth. In one respect such growth is simpler than multi-sector national growth. In international trade theory usually neither labor nor capital are free to move internationally. To Ricardo such lack of factor mobility was the very rationale for building a separate theory of international trade. To us, such lack of factor mobility is a welcome simplification: We need to worry about the allocation of neither labor nor capital among countries. We do need to worry about the allocation of goods among countries. This problem we

shall simplify by assuming the existence of merely two countries, each producing a separate good. A country's good may be invested or consumed. If invested it is not traded internationally but is installed as immortal capital stock in its country of origin. If consumed it may be traded internationally. Each country's consumers consider the two goods good but not perfect substitutes and will consume some of each good. How much will depend upon prices, the exchange rate, and consumer income.

In another respect international growth is more complicated than national growth. The second rationale for a separate theory of international trade was the existence of different monetary units in different countries. This will add another variable to our system, i. e., the price of one monetary unit in terms of the other. But it will also add another equation to our system, i. e., that a flexible exchange rate will equilibrate the balance of payments.

Much international trade theory is static and lies beyond the scope of the present book. Let us at least build a dynamic mini model of international trade admitting three items often ignored even in the few such models that do exist,[1] i. e., consumer preferences, disparity

of rates of technological progress, and disparity of rates of growth of labor forces.

I. NOTATION

Variables

C_{ij} ≡ physical consumption in jth country of goods produced in ith country

D ≡ demand for money

E ≡ exchange rate in number of monetary units of Country 1 exchanged for one monetary unit of Country 2

g_v ≡ proportionate rate of growth of variable v ≡ C, D, E, I, κ, L, M, P, r, ρ, S, w, X, and Y

I ≡ physical investment

κ ≡ physical marginal productivity of capital stock

L ≡ labor employed

λ ≡ proportion employed of available labor force

M ≡ supply of money

P ≡ price of goods

p ≡ multiplicative factor of Phillips curve representing inflationary potential

r ≡ nominal rate of interest

ρ ≡ real rate of interest

S ≡ physical capital stock

U ≡ utility

w ≡ money wage rate

X ≡ physical output

Y ≡ money income

Parameters

a ≡ multiplicative factor of production function

α, β ≡ exponents of production function

c ≡ propensity to consume money income

F ≡ available labor force

g_v ≡ proportionate rate of growth of parameter $v \equiv a$ and F

m ≡ multiplicative factor of demand for money function

μ ≡ exponent of demand for money function

π ≡ exponent of Phillips function

u ≡ multiplicative factor of utility function

υ ≡ exponent of utility function

Subscripts i and j refer to country number. All parameters are stationary except a and F whose growth rates are.

II. A MINI MODEL OF INTERNATIONAL GROWTH

1. Definitions

Define the proportionate rate of growth

$$(1) \qquad g_v \equiv \frac{dv}{dt}\frac{1}{v}$$

Define investment in the ith country as the derivative of capital stock with respect to time:

$$I_i \equiv \frac{dS_i}{dt} \tag{2}$$

2. Production

Let entrepreneurs of the ith country apply a Cobb-Douglas production function

$$X_i = a_i L_i^{\alpha_i} S_i^{\beta_i} \tag{3}$$

where $0 < \alpha_i < 1$; $0 < \beta_i < 1$; $\alpha_i + \beta_i = 1$; and $a_i > 0$.

In each country let profit maximization under pure competition equalize real wage rate and physical marginal productivity of labor:

$$(4) \qquad \frac{w_i}{P_i} = \frac{\partial X_i}{\partial L_i} = \alpha_i \frac{X_i}{L_i}$$

Physical marginal productivities of capital stock are

$$(5) \qquad \kappa_i \equiv \frac{\partial X_i}{\partial S_i} = \beta_i \frac{X_i}{S_i}$$

3. Investment Demand

Let N_i be the present net worth of new capital stock S_i installed by an entrepreneur in the ith country. Let his desired capital stock be the size of stock maximizing present net worth. Use the stationary nominal rate of interest r_i as a discount rate. Define present gross worth of another physical unit of capital stock as the present worth of all its future value marginal productivities over its entire useful life. Find present net worth of another physical unit of capital stock as its

gross worth minus its price and set it equal to zero, as we did for a closed economy in Chapter 6, Sec. II, 3. Desired capital stock will then be

$$S_i = \beta_i X_i / \rho_i \tag{6}$$

where

$$\rho_i \equiv r_i - (g_{Ki} + g_{Pi}) \tag{7}$$

Apply the definitions (1) and (2) to (6) and find desired investment as the derivative of desired capital stock with respect to time:

$$I_i \equiv \frac{dS_i}{dt} = \beta_i g_{Xi} X_i / \rho_i \tag{8}$$

Eqs. (6) and (8) are capital stock and investment desired by an individual entrepreneur. Except X_i everything on the right-hand sides of (6) and (8) is common to all entrepreneurs of the ith country.

Factor out all common factors and sum over all entrepreneurs, then X_i becomes national output, and (6) and (8) become national desired capital stock and investment.

4. Consumption Demand

And now for our only international allocation problem, the allocation of consumers' goods between countries. Within each country let all consumers have the same utility function and propensity to consume, but let both differ between countries. In the jth country let every consumer have the utility function

$$U_j = u_j C_{1j}^{\upsilon_{1j}} C_{2j}^{\upsilon_{2j}}$$

where $0 < \upsilon_{ij} < 1$, and $u_j > 0$. Let every consumer spend the fraction c_j where $0 < c_j < 1$, of his money income Y_j. Then his budget constraint is

$$c_1 Y_1 = P_1 C_{11} + E P_2 C_{21}$$

$$c_2 Y_2 = P_1 C_{12}/E + P_2 C_{22}$$

In the jth country maximize the consumer's utility subject to his budget constraint and find his two demand functions C_{1j} and C_{2j}

$$C_{11} = c_{11} Y_1 / P_1$$

$$C_{12} = c_{12} Y_2 / (P_1/E)$$

$$C_{21} = c_{21} Y_1 / (EP_2)$$

$$C_{22} = c_{22} Y_2 / P_2$$

where $c_{ij} \equiv c_j \upsilon_{ij}/(\upsilon_{1j} + \upsilon_{2j})$. Here is consumption desired by an individual consumer in the jth country. Except Y_j everything on the right-hand sides is common to all consumers of the jth country. Factor out all common factors, sum over all consumers of the jth country, then Y_j becomes national money income of the jth country, and C_{ij} becomes national desired consumption. But with immortal capital stock, the

5. Goods-Market Equilibrium

In each country, goods-market equilibrium requires the supply of goods to equal the sum of consumption, export, and investment demand for them:

$$X_i = \sum_{j=1}^{2} C_{ij} + I_i \tag{14}$$

6. Employment and the Phillips Function

In the ith country let labor employed be the proportion λ_i of available labor force, where $0 < \lambda_i < 1$, and λ_i is so far not a function of time:

$$L_i = \lambda_i F_i \tag{15}$$

Subtract employment (15) from available labor force F_i, find unemployment $(1 - \lambda_i)F_i$, and express a Phillips curve

entire value of national output represents value added, i. e., national money income

$$Y_j \equiv P_j X_j \tag{9}$$

Insert (9) and write national desired consumption as the four Graham (1923) demand functions

$$C_{11} = c_{11} X_1 \tag{10}$$

$$C_{12} = c_{12} P_2 X_2 / (P_1/E) \tag{11}$$

$$C_{21} = c_{21} P_1 X_1 / (E P_2) \tag{12}$$

$$C_{22} = c_{22} X_2 \tag{13}$$

Graham demand functions have income and price elasticities of 1 and -1, respectively. Our appendix reproduces a few empirical income and price elasticities of demand.

(16) $$g_{wi} = p_i(1 - \lambda_i)^{\pi_i}$$

where $\pi_i < 0$, $p_i > 0$, and p_i———like λ_i———is so far not a function of time. The multiplicative factor p_i determines the level of the Phillips curve and represents the inflationary potential of the ith country.

7. Money

In the ith country let the demand for money be a function of national money income and the nominal rate of interest:

(17) $$D_i = m_i Y_i r_i^{\mu_i}$$

where $\mu_i < 0$, and $m_i > 0$.

Money-market equilibrium requires the supply of money to equal the demand for it:

(18) $$M_i = D_i$$

8. Balance-of-Payments Equilibrium

Balance-of-payments equilibrium requires Country 1's consumption of Country 2's goods to equal Country 2's consumption of Country 1's goods, both being measured in the same monetary unit, or exchange reserves of the ith country would either accumulate or be depleted. Thus

$$EP_2C_{21} = P_1C_{12} \tag{19}$$

Insert our Graham demand functions (11) and (12) into our balance-of-payments equilibrium (19) and express the exchange rate as

$$E = c_{21}P_1X_1/(c_{12}P_2X_2) \tag{20}$$

III. STEADY-STATE EQUILIBRIUM GROWTH-RATE SOLUTIONS

1. Underemployment Inflationary Steady-State Equilibrium Growth

By taking derivatives with respect to time of all equations involving the variables C_{ij}, D_i, E, I_i, κ_i, L_i, M_i, P_i, r_i, ρ_i, S_i, w_i, X_i, and Y_i the reader may convince himself that the system (1) through (19) is satisfied by the following growth-rate solutions.

(21) $g_{Cij} = g_{Xi}$

(22) $g_{Di} = g_{Mi}$

(23) $g_E = g_{P1} + g_{X1} - (g_{P2} + g_{X2})$

(24) $g_{Ii} = g_{Xi}$

(25) $g_{\kappa i} = 0$

(26) $g_{Li} = g_{Fi}$

(27) $g_{Mi} = g_{Yi}$

(28) $g_{Pi} = g_{wi} - g_{ai}/\alpha_i$

(29) $g_{ri} = 0$

(30) $g_{\rho i} = 0$

(31) $g_{Si} = g_{Xi}$

(32) $g_{wi} = p_i(1 - \lambda_i)^{\pi_i}$

(33) $g_{Xi} = g_{ai}/\alpha_i + g_{Fi}$

(34) $g_{Yi} = g_{Fi} + g_{wi}$

International steady-state equilibrium growth is consistent with unemployment ($0 < \lambda_i < 1$) and inflation ($p_i > 0$). Our growth was steady-state growth, for no right-hand side of our growth-rate solutions (21) through (34) was a function of time. Our growth was equilibrium growth implying, first, that supply equals demand. Here our conditions (14) and (18) were met. Second, equilibrium implies self-fulfilling expectations. Here we used the same symbol for the planned, expected, and realized value of any variable, thus implying the equality of the three. But is such equality always possible? Yes if the system has a set of solutions. No if the system has no such set. Our own system did have the set of solutions (21) through (34). Consequently, self-fulfilling expectations were possible, and our growth was equilibrium growth.

2. Is Growth Balanced?

By balanced growth we mean identical proportionate rates of growth of physical output of all goods. In (33) neither the rates of technological progress g_{ai}, the labor elasticities of output α_i, nor the rates of growth of labor force g_{Fi} need be the same in the two countries. Growth may be balanced but only as an odd piece of luck!

It does, however, follow from (23), (28), and (33) that, when measured in Country 1's monetary unit, the money value of output P_1X_1 and EP_2X_2 of the two countries will be growing at the common rate $g_{F1} + p_1(1 - \lambda_1)^{\pi_1}$.

3. What Difference Do λ_i and p_i Make in our Growth-Rate Solutions?

Some of our solutions had in them the employment fraction λ_i and the multiplicative factor p_i of the Phillips curve representing the inflationary potential. As a result, our system simply has infinitely many solutions, i. e., for a given employment fraction λ_i one for each value of the inflationary potential p_i, and for a given value of the inflationary potential p_i one for each employment fraction λ_i. But in our growth-rate solutions what difference do λ_i and p_i make?

Well, we find g_{wi} and with it λ_i and p_i to be absent from the growth-rate solutions for the eight variables C_{ij}, I_i, κ_i, L_i, r_i, ρ_i, S_i, and X_i. Is g_{wi} and with it λ_i and p_i also absent from the growth-rate solutions for the real wage rate———as it was in our closed economy of Chapter 6?

4. Rates of Growth of Real Wage Rates

In each country there are now two real wage rates, i. e., one in terms of the country's own good and one in terms of the imported good.

The ith country's real wage rate in terms of that country's own good is w_i/P_i. Using our solution (28) we find the growth rate of that real wage rate to be

$$g_{wi} - g_{Pi} = g_{ai}/\alpha_i \tag{35}$$

The first country's real wage rate in terms of the second country's good is $w_1/(EP_2)$. Using our solutions (23), (28), and (33) we find the growth rate of that real wage rate to be

$$g_{w1} - (g_E + g_{P2}) = g_{a2}/\alpha_2 + g_{F2} - g_{F1} \tag{36}$$

The second country's real wage rate in terms of the first country's good is $w_2/(P_1/E)$. Again using (23), (28), and (33) we find the growth rate of that real wage rate to be

$$(37) \qquad g_{w2} - (g_{P1} - g_E) = g_{a1}/\alpha_1 + g_{F1} - g_{F2}$$

In the growth rates of all real wage rates (35) through (37), then, g_{wi} and with it λ_i and p_i will cancel. In other words, each growth rate will be the same for any value of the employment fraction λ_i and for any value of the inflationary potential p_i. Consequently, Friedman's (1968) "natural" rate of unemployment is not unique in our international economy either!

We may summarize our results (35) and (37) as follows. The growth rate of the ith country's real wage rate in terms of that country's own good depends solely upon that country's own productivity growth g_{ai} and labor exponent α_i: A country's labor is the better off the more rapidly its own productivity is growing. The growth rate of the ith country's real wage rate in terms of the jth country's good depends upon the jth country's productivity growth g_{aj} and labor exponent α_j as well as upon the difference $g_{Fj} - g_{Fi}$ between the rates of growth of the jth and ith country's labor forces. In other words, a country's labor is the better off the more rapidly productivity and labor force is growing in the <u>other</u> country and the less rapidly the country's own labor force is growing.

5. Rate of Growth of the Exchange Rate

We find g_{wi} and with it λ_i and p_i to be present in the growth-rate solutions for the remaining six variables, i. e., the exchange rate E, money demand D_i and supply M_i, price P_i, money wage rate w_i, and national money income Y_i. Of special interest in the present chapter is the growth rate of the exchange rate.

Inserting our solutions (28), (32), and (33) into (23) we find the rate of growth of the exchange rate

$$g_E = p_1(1 - \lambda_1)^{\pi_1} - p_2(1 - \lambda_2)^{\pi_2} + g_{F1} - g_{F2} \tag{38}$$

So g_{wi} and with it λ_i and p_i of <u>both</u> countries is present in the growth-rate solution for the exchange rate. Specifically: Since $\pi_i < 0$, a high employment fraction in the first country λ_i will make g_E high, whereas a high employment fraction in the second country λ_2 will make g_E low. A high inflationary potential in the first country p_1 will make g_E high, whereas a high inflationary potential in the second country p_2 will make g_E low.

6. The Purchasing Power Parity Theory of the Exchange Rate

The purchasing power parity theory of the exchange rate, formulated in the wake of First-World-War inflation, said that the exchange rate would be in direct proportion to the price level of the first country and in inverse proportion to the price level of the second country. Reformulate it in terms of growth rates, and it says that the rate of growth of the exchange rate will be the rate of growth of price in the first country minus the rate of growth of price in the second country.

That isn't what our solution (23) says. We have found the rate of growth of the exchange rate to be the sum of the rates of growth of price and physical output in the first country minus the sum of the rates of growth of price and physical output in the second country. The purchasing power parity theory ignores the rates of growth of the physical outputs! Why? Pre-Keynesian theory was more price-conscious than output-conscious. But in all fairness, at the time the purchasing power parity theory was formulated and applied, rates of growth of physical outputs were swamped by the rates of growth of prices. And Yeager (1958) successfully applied the theory even to the period 1937-1957. But in principle,

we must insist, the exclusion of the rates of growth of the physical outputs is inadmissible.

IV. STEADY-STATE EQUILIBRIUM INTEREST-RATE SOLUTIONS

1. The Levels of the Interest Rates

Each country has two interest rates. First there is a real rate of interest ρ_i found by inserting our solution (25) into our definition (7):

$$\rho_i = r_i - g_{Pi} \tag{39}$$

To solve for ρ_i, insert first our balance-of-payment equilibrium condition (19), then insert our investment demand (8) and our Graham demand functions (10) through (13) into our goods-market equilibrium condition (14), and use the definition of v_{ij} to see that

$$c_{11} + c_{21} \equiv c_1$$

$$c_{12} + c_{22} \equiv c_2$$

and find the solutions for the real rates of interest

$$\rho_i = \beta_i g_{Xi}/(1 - c_i) \tag{40}$$

in complete analogy with our result (39) in Chapter 6 for a closed economy.

To solve for the nominal rate of interest r_i insert (28), (32), (33), and (40) into (39) and find

$$r_i = [(c_i - \alpha_i)g_{ai}/\alpha_i + \beta_i g_{Fi}]/(1 - c_i) + p_i(1 - \lambda_i)^{\pi_i} \tag{41}$$

in complete analogy with our result (40) in Chapter 6 for a closed economy.

To uphold the values (40) of ρ_i and (41) of r_i, what would the rate of growth g_{Mi} of the money supply have to be? Insert (32) and (34) into (27) and find

$$g_{Mi} = g_{Fi} + p_i(1 - \lambda_i)^{\pi_i} \tag{42}$$

in complete analogy with our result (41) in Chapter 6 for a closed economy.

2. What Difference do λ_i and p_i Make in our Interest-Rate Solutions?

Do our interest-rate solutions have in them the employment fraction λ_i and the multiplicative factor p_i representing the inflationary potential?

We find both λ_i and p_i absent from our solution (40) for the real rate of interest ρ_i. Upholding (40), then, would be compatible with any value of the employment fraction λ_i or the inflationary potential p_i. Upholding (40) would guarantee neither full employment nor absence of inflation———again the analogy to the closed economy is complete.

By contrast, we find both λ_i and p_i present in our solutions (41) for the nominal rate of interest r_i as well as (42) for the rate of growth g_{Mi} of required money supply. Both are the higher, the higher are the employment fraction λ_i and the inflationary potential p_i.

Summarizing, then, each level of the two real——but not of the two nominal—— rates of interest will be the same for any value of the employment fraction and for any value of the inflationary potential. Does this mean that employment and inflation can both be controlled by monetary policy? The answer is No, as we shall now see.

3. Monetary Policy

Allow the employment fraction λ_i and the inflationary potential p_i to vary with time. In an effort to raise the former, let the monetary authorities permit the money supply to grow at a rate higher than its steady-state equilibrium value (42). At so far unchanged expectation g_{pi}, the nominal and real rate of interest, r_i and ρ_i, will then fall below their equilibrium levels (41) and (40), respectively. As a result, our goods-market equilibrium condition (14) will now be violated, and positive excess demand will emerge. Positive excess demand might stimulate employment by raising λ_i.

In an effort to reduce the inflationary potential p_i, let the monetary authorities force the money supply to grow at a rate lower than its steady-state equilibrium value (42). At so far unchanged expectations

g_{Pi}, the nominal and real rate of interest, r_i and ρ_i, will now rise above their equilibrium levels (41) and (40). As a result, negative excess demand will emerge. Negative excess demand might decelerate inflation by lowering p_i.

The dilemma of public policy is neatly illustrated by the fact that the two public policies sketched are mutually exclusive!

V. CONCLUSIONS

We have internationalized our growth model———but sparingly. Neither labor nor capital move internationally, only goods do. And goods move only in the form of consumers' goods. International demand functions are of the simplest possible, i. e., Graham, form with income and price elasticities of 1 and -1, respectively.

Under such assumptions we found simple solutions for underemployment inflationary steady-state equilibrium growth. By and large, the theoretical results of Chapter 6 for a closed economy still stand: Each rate of growth of the four real wage rates will be the same for

any value of the employment fraction and for any value of the inflationary potential. As a result, Friedman's "natural" rate of unemployment is not unique in our international economy either. Each level of the two real——but not of the two nominal——rates of interest will be the same for any value of the employment fraction and for any value of the inflationary potential. Employment and inflation cannot both be controlled by monetary policy.

But we also found some specifically international results. International growth will be balanced only as an odd piece of luck. The purchasing power parity theory of the exchange rate was not the whole story: The rate of growth of the exchange rate depends upon the rates of growth of physical outputs no less than upon the rates of growth of prices. Ultimately it was found to depend upon the employment fractions and the inflationary potentials of both countries.

A P P E N D I X

EMPIRICAL INCOME AND PRICE ELASTICITIES OF DEMAND

Graham demand functions have income and price elasticities of 1 and -1, respectively. Like ourselves, Graham used his demand functions for theoretical convenience and made no attempt at empirical estimation. Empirical elasticities of United States import with respect to income were estimated at 0.91 and 1.6 by Ball and Mavwah (1962); at 1.00 by Harberger (1957); at 1.51 by Houthakker and Magee (1969); at 1.27 by Kreinin (1967); and at 1.94, 1.43, and 0.96 by Murray and Ginman (1976).

Elasticities of United States import with respect to price were estimated at -0.51 and -1 by Ball and Mavwah (1962); at -0.95 by Harberger (1957); at -0.54 by Houthakker and Magee (1969); at -1.1 by Kreinin (1967); and at -1.23, -0.71, and -1.05 by Murray and Ginman (1976).

Income and price elasticities of the import of 14 other major countries were estimated at 1.37 and -0.90, respectively, by Houthakker and Magee (1969).

FOOTNOTE

[1]E. g., Bardhan (1965) ignored all three items.

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